

# Pushing the Bounds of Numerical Ranges

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## Abstract

The numerical range, also known as the field of values, has applications in physics and quantum computing; our own research is beyond cutting-edge, and will most likely find its application decades in the future. Let  $T(x)$  be the conjugate transpose of  $x$ , where  $x$  is a complex unit vector. The numerical range of a matrix  $M$  is the mapping of  $x$  to the set of all numbers of the form  $T(x)Mx$ . An integer lattice point is defined as an ordered pair of integer coordinates. We desire to find invariant upper bound formulas for the area of and lattice points contained within the numerical range of any given matrix. We begin by finding the exact area of the numerical range of a 2-by-2 matrix, whose shape is an ellipse and whose area is easily calculable. We then look into upper area bounds for any size matrix. The strategy by which this is accomplished is circumscribing the numerical range in a box. Motivated by a desire to find a universal bound, we work to translate the numerical range to contain the origin. We use a circular area bound to improve our earlier rectangular bound, and provide an easily calculable upper bound for the radius. Thus, we arrive at the author's new proof of an upper bound for the area of the numerical range of any matrix, given by a formula in terms of the trace of our matrix and thus easily calculable directly from the entries of the matrix. We also use the trace of the matrix to provide an upper bound on the number of lattice points contained within the numerical range.